

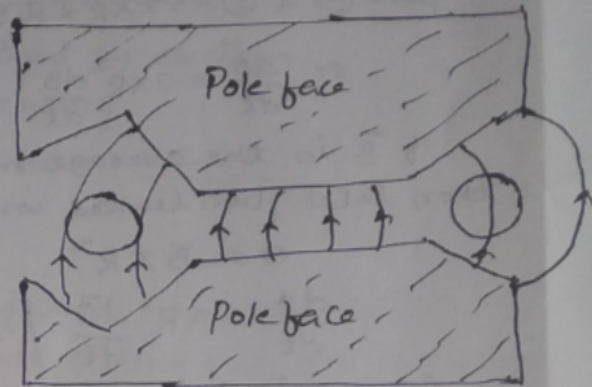
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Q Describe the construction and working of Betatron?

Ans Betatron:-

Betatron involves a device that is free from the difficulty of relativistic variation of mass with velocity. It employs no electric field, but the magnetic field is itself alternating and produces an inductive electromotive force on the circulating electrons. Thus electrons are accelerated and hence a Betatron is also called as induction accelerator.

It consists of a doughnut shaped vacuum chamber made of glass. The inner side is coated with thin layer of Ag to avoid the accumulation of surface charges. The



electromagnet is also doughnut shaped made up of laminated iron to eddy currents. The magnet produces a strong magnetic field in the doughnut. The electrons are produced by the electron gun and are allowed to move in a circular orbit of constant radius in the vacuum chamber. The magnetic flux linked with the orbit changes with time and electrons gain energy by induction. The principle of a betatron can be explained by considering the electron of mass m and velocity ' v ' moves in a circular path having radius R where magnetic field ' B ' acts at right angle

$$\therefore Bev = \frac{mv^2}{R} \quad \text{--- (1)}$$

$$BeR = mv = p = \text{momentum of electron}$$

A flux ϕ cuts this orbit and is changed at the rate of $d\phi/dt$. Thus an induced e.m.f is produced

causing more speedy motion of these electrons.

Let the induced electric field compo tangential to the orbit E .

$$\therefore \text{Induced e.m.f } 2\pi R \cdot E = \frac{d\phi}{dt} \quad \text{--- --- ---}$$

The force acting on the electron (eE) will be eq the rate of change of its momentum

$$\therefore eE = \frac{dp}{dt}$$

$$\Rightarrow eE = \frac{d}{dt} (BeR)$$

$$\therefore E = R \frac{dB}{dt}$$

$$\text{from } \textcircled{2} \text{ \& } \textcircled{3} \quad 2\pi R \times R \frac{dB}{dt} = \frac{d\phi}{dt}$$

$$\text{or } \frac{d\phi}{dt} = 2\pi R^2 \frac{dB}{dt}$$

If \bar{B} is the average magnetic field over the area then total flux linked with the orbit is

$$\phi = \bar{B} \pi R^2$$

$$\frac{d\phi}{dt} = \pi R^2 \frac{d\bar{B}}{dt}$$

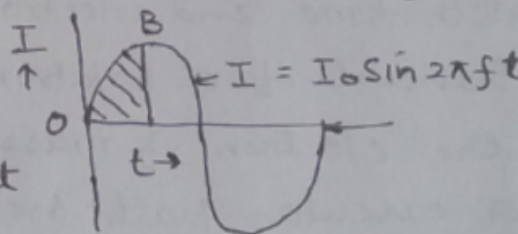
from eqⁿ $\textcircled{4}$ & $\textcircled{5}$

$$\pi R^2 \frac{d\bar{B}}{dt} = 2\pi R^2 \frac{dB}{dt}$$

$$\therefore 2 \frac{dB}{dt} = \frac{d\bar{B}}{dt}$$

Eqⁿ $\textcircled{7}$ represents Betatron. It says that the rate of change of flux density within the orbit ($\frac{d\bar{B}}{dt}$) must equal to twice the rate of change of flux density at

In order to create flux changes, a sinusoidal varying current is introduced into the coil of the electromagnet during the first one quarter of



the cycle. Shaded portion of the curve. After the point the current is disconnected from the electromagnet and connected to a dummy load for rest $\frac{3}{4}$ th of the cycle.

Energy of electron :- change in flux takes place

③

during $\frac{1}{4}$ th of cycle (i.e. $T = \frac{1}{f}$, $\frac{T}{4} = \frac{1}{4f}$). Electrons travel with almost velocity of light c . Total distance travelled in $T/4$ period is equal to the $c/4f$. The number of revolutions

$$N = \frac{\text{Total distance travelled}}{\text{Circumference of orbit}} = \frac{c/4f}{2\pi R} = \frac{c}{8\pi Rf}$$

The flux variation is given by $\phi = \phi_0 \sin 2\pi ft$

The energy gained by electron per unit will be

$$eE = e \frac{d\phi}{dt} = e \cdot 2\pi f \phi_0 \cdot \cos 2\pi ft$$

Average value of over quarter period ($T/4$)

$$= 2\pi e \phi_0 \left(\frac{2}{\pi}\right) = 4\pi f e \phi_0$$

Hence total energy gained by electron

$$E = N \times \text{Average energy}$$

$$= \frac{c}{8\pi R} \times 4\pi f e \phi_0 = \frac{ce\phi_0}{2\pi R}$$

Discussion:-

- ① When the magnetic field has reached its max^m value, the electrons are pulled out the orbit by an auxillary coil through which current is passed. If this not done, magnetic field starts decreasing and electron will start standing down.
- ② The high energy electrons from betatron have been used to produce X-rays.
- ③ The upper limit of energy attained by electrons with betatron is 500 MeV
- ④ The only disadvantage of betatron is that large magnet is needed to supply the variable magnetic flux